

# On the detection of condensate vortex states

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We discuss a detection scheme that reveals the existence of vortex states in a cylindrically symmetric condensate trap. It relies on the measurement of the second-order correlation function of the Schrödinger field, and yields directly the topological charge of the vortex state.

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The rapid progress in the experimental generation and manipulation of Bose-Einstein condensates in low density trapped alkali vapors [1–5] opens up the way to the detailed study of the thermodynamic and dynamical properties of weakly interacting quantum-degenerate gases. A topic of much current interest is the superfluidity of these samples. Superfluidity is related to the rotational properties, and in particular to the existence of vortex states in quantum gases [6]. While no vortex states have been launched in low density atomic condensates so far, their existence has been numerically studied [7–9] and their stability analyzed [10]. Optical methods to launch vortex states were recently proposed in Ref. [11,12], and Jackson et. al. [13] have produced numerical solutions of vortex formation in a weakly-interacting condensate by piercing it and subsequently slicing it with a blue-detuned laser at a velocity exceeding the critical velocity.

In view of these developments, it is therefore timely and important to discuss ways to detect vortices in trapped atomic condensates. One possibility suggested in Ref. [11] is based on measuring the spatial absorption profile of the sample. This method is sensitive to the rotation of the condensate via the rotational Doppler shift [14], but does not give a direct measure of its topological charge. Rather, the quantization of the vortex motion must be inferred from other condensate parameters such as number of particles, system size, etc. In contrast, the scheme that we propose and analyze in this note produces a spatial interference pattern that results directly from the nonzero atomic angular momentum of the vortex and gives a direct measurement of its topological charge. In addition, a slight variation of that method allows a direct demonstration of the existence of persistent currents in the sample. The method is based on the ionization detection scheme discussed in [15], whereby one measures normally ordered correlation functions of the Schrödinger field operator  $\hat{\Psi}(\mathbf{r}, t)$ . Specifically, we show that the two-point second-order correlation functions provide direct information on the persistent currents and the topological charge of the vortex state.

The proposed schemes [11,12] to create vortices in cylindrically symmetric traps rely on Raman transitions between two hyperfine levels of ground state alkali atoms, the effective coupling resulting from virtual electric dipole transitions induced by far-off resonant laser

beams. For this discussion we assume that the system is effectively two-dimensional as a result of strong confinement in the  $z$ -direction. A coherent rotational coupler is realized if the lasers used for the Raman transitions carry angular momentum associated with their field envelope, which can be transferred to the center-of-mass motion of the atoms. This can be achieved e.g. [11] by placing the condensate at the focus  $z = 0$  of two Laguerre-Gaussian beams of polarizations  $\sigma^+$  and  $\sigma^-$  propagating along the  $z$  direction and with Rabi frequencies

$$\Omega_{\pm}(\mathbf{r}) = \Omega_0 \left( \sqrt{2} \frac{\rho}{w} \right)^{|m|} L_p^{|m|} \left( 2 \frac{\rho^2}{w^2} \right) e^{-\rho^2/w^2} e^{\pm im\varphi} e^{ikz} \quad (1)$$

where  $\rho^2 = x^2 + y^2$ ,  $w$  is the beam spot size, and  $\varphi$  is the azimuthal angle. The transverse structure of these fields is characterized by the radial mode number  $p$  and the azimuthal mode number  $m$  [16]. In particular, they have a phase singularity at their center with topological charge  $\pm m$  and associated angular momentum  $\pm \hbar m$ . As a result of the Raman transitions the laser fields couple a condensate in the ground state  $\langle \mathbf{r} | g \rangle = \psi_g(\mathbf{r}) | F = 1, M_F = -1 \rangle$  with no angular momentum, where  $\psi_g(\mathbf{r})$  is the ground state wave function of the trapping potential, to the vortex state  $\langle \mathbf{r} | v \rangle = \psi_v(\mathbf{r}) | F = 1, M_F = 1 \rangle$  with  $2\hbar m$  angular momentum per atom and topological charge  $2m$ . For the geometry discussed in Ref. [11] and using  $m = 1$  the vortex wave function is given explicitly by

$$\psi_v(\mathbf{r}) = \mathcal{N} \rho^2 e^{i\theta(\mathbf{r})} \psi_g(\rho), \quad (2)$$

with  $\mathcal{N}$  a normalization constant, and  $\theta(\mathbf{r}) = 2\varphi$ . The key point from this discussion is that the vortex state created in this manner has a topological charge  $2m$  and associated azimuthal variation  $\exp(2im\varphi)$  in the general case.

Since  $\langle v | g \rangle = 0$ , this system can conveniently be described in terms of the spinor  $\phi(\mathbf{r}, t) = \text{col}(\phi_g(\mathbf{r}, t), \phi_v(\mathbf{r}, t))$ . Then in the mean-field Hartree approximation, and assuming that many-body effects do not significantly alter the spatial profiles of the ground state and vortex state so that we may restrict ourselves to these two modes, the components of the spinor evolve according to

$$\phi_v(\mathbf{r}, t) = \beta(t)\psi_v(\mathbf{r}), \quad \phi_g(\mathbf{r}, t) = \alpha(t)\psi_g(\mathbf{r}). \quad (3)$$

For the case  $m = 1$   $\alpha(t)$  and  $\beta(t)$  are governed by the equations of motion (5) and (6) of Ref. [11], and this serves as a concrete example.

More generally, in the following we consider a generic situation where the external perturbation responsible for the coupling creates a quantum degenerate gas in the superposition (3), but with the phase  $\theta(\mathbf{r})$  of the excited state in Eq. (2) kept general. This will allow us to point out the specific signatures of a vortex state that distinguish it from a non-vortex state.

In the two-mode approximation the condensate is described by the two-component Schrödinger field operator  $\hat{\Psi}(\mathbf{r}) = (\hat{\Psi}_g(\mathbf{r}), \hat{\Psi}_v(\mathbf{r}))$  satisfying the boson commutation relation  $[\hat{\Psi}_\ell(\mathbf{r}), \hat{\Psi}_\ell^\dagger(\mathbf{r}')] = \delta_{\ell\ell'}\delta(\mathbf{r}-\mathbf{r}')$ ,  $\ell = \{v, g\}$ . Assuming then a system composed of  $N$ -atoms, the quantum state of the system becomes

$$|\Phi(t)\rangle = \frac{1}{\sqrt{N!}} \left[ \int d^3r (\phi_g(\mathbf{r}, t) \hat{\Psi}_g^\dagger(\mathbf{r}) + \phi_v(\mathbf{r}, t) \hat{\Psi}_v^\dagger(\mathbf{r})) \right]^N |0\rangle, \quad (4)$$

or, in terms of the mode creation and annihilation operators

$$a_\ell^\dagger = \int d^3r \psi_\ell(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}), \quad a_\ell = \int d^3r \psi_\ell^*(\mathbf{r}) \hat{\Psi}(\mathbf{r}), \quad (5)$$

we obtain

$$|\Phi(t)\rangle = \frac{1}{\sqrt{N!}} \sum_k C_N^k [\alpha(t)]^k [\beta(t)]^{N-k} (a_g^\dagger)^k (a_v^\dagger)^{N-k} |0\rangle. \quad (6)$$

Hence, the sample is in an entangled superposition of the ground and vortex states, a result of the fact that the total number of particles is conserved, but not the individual particle numbers in the two states. This is similar to the situation of split condensates discussed in Refs. [17,18].

Due to the assumed cylindrical symmetry of the trapping potential, the existence of a vortex state cannot be demonstrated by off-resonance imaging,<sup>1</sup> which measures correlation functions of the sample density  $\hat{\rho}(\mathbf{r}, t) \equiv \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t)$ . Since this is simply the sum of the condensate and vortex density profiles, and the vortex density profile is cylindrically symmetric, the density profile does not reveal the phase singularity associated with the vortex. What is needed instead is a measurement scheme which involves correlation functions of  $\hat{\Psi}(\mathbf{r}, t)$  itself. As

discussed in Ref. [15], these functions can be extracted in an ionization scheme whereby one or more tightly focused lasers are used to selectively ionize atoms in small regions of the condensate plus vortex system. The measurement proceeds then by detecting the ionized atoms, which play the role of a detector field.

We consider specifically a two-point detection scheme in which two ionizing laser beams are focussed at locations  $\mathbf{r}_1 = (\rho_1, \varphi_1)$  and  $\mathbf{r}_2 = (\rho_2, \varphi_2)$ . For that geometry, and assuming that the lasers are focussed onto spots small compared to the dimensions of the condensate, the ionization scheme measures the probability  $w_2$  of jointly ionizing an atom at  $\mathbf{r}_1$  and the other at  $\mathbf{r}_2$  as a function of normally ordered Schrödinger field correlation functions whose explicit form is

$$\begin{aligned} w_2(t, \Delta t) \simeq & \eta(\mathbf{r}_1, \mathbf{r}_2) \eta(\mathbf{r}_2, \mathbf{r}_1) \int_t^{t+\Delta t} dt_1 \int_t^{t+\Delta t} dt_2 \\ & \langle \hat{\Psi}^\dagger(\mathbf{r}_1, t_1) \hat{\Psi}^\dagger(\mathbf{r}_2, t_2) \hat{\Psi}(\mathbf{r}_2, t_1) \hat{\Psi}(\mathbf{r}_1, t_2) \rangle \\ & + \eta(\mathbf{r}_1) \eta(\mathbf{r}_2) \int_t^{t+\Delta t} dt_1 \int_t^{t+\Delta t} dt_2 \\ & \langle \hat{\Psi}^\dagger(\mathbf{r}_1, t_1) \hat{\Psi}^\dagger(\mathbf{r}_2, t_2) \hat{\Psi}(\mathbf{r}_2, t_2) \hat{\Psi}(\mathbf{r}_1, t_1) \rangle, \quad (7) \end{aligned}$$

where  $\eta(\mathbf{r})$  is the detector efficiency and  $\eta(\mathbf{r}_1, \mathbf{r}_2)$  its cross-efficiency, as discussed in Ref. [15]. The first term in  $w_2$  is an exchange contribution, while the second term is the direct contribution familiar from photodetection theory.

Equation (7) can be considerably simplified for measurement intervals  $\Delta t$  small compared to the characteristic evolution time of the condensate. In this case we can set  $t_1 = t_2 = t$  and we have simply

$$\begin{aligned} w_2(t) = & (\Delta t)^2 [\eta(\mathbf{r}_1) \eta(\mathbf{r}_2) + \eta(\mathbf{r}_1, \mathbf{r}_2) \eta(\mathbf{r}_2, \mathbf{r}_1)] G^{(2)}(\mathbf{r}_1, \mathbf{r}_2; t), \quad (8) \end{aligned}$$

where

$$\begin{aligned} G^{(2)}(\mathbf{r}_1, \mathbf{r}_2; t) \equiv & \langle \Phi(0) | \hat{\Psi}^\dagger(\mathbf{r}_1, t) \hat{\Psi}^\dagger(\mathbf{r}_2, t) \hat{\Psi}(\mathbf{r}_2, t) \hat{\Psi}(\mathbf{r}_1, t) | \Phi(0) \rangle \\ = & \langle \Phi(t) | \hat{\Psi}^\dagger(\mathbf{r}_1) \hat{\Psi}^\dagger(\mathbf{r}_2) \hat{\Psi}(\mathbf{r}_2) \hat{\Psi}(\mathbf{r}_1) | \Phi(t) \rangle. \quad (9) \end{aligned}$$

As follows from the definitions of the detector self- and cross-efficiencies  $\eta(\mathbf{r})$  and  $\eta(\mathbf{r}_1, \mathbf{r}_2)$  given in Ref. [15] the term in square brackets in Eq. (8) does not vary azimuthally, which is important for the following considerations. Specializing in anticipation of our subsequent discussion to the case  $\rho_1 = \rho_2 = \rho$  and with Eq. (5) this gives readily

$$\begin{aligned} G^{(2)}(\mathbf{r}_1, \mathbf{r}_2; t) = & 2N(N-1) |\phi_g(\rho)|^2 |\phi_v(\rho)|^2 |\alpha(t)|^2 |\beta(t)|^2 \\ & \times (1 + \cos[\theta(\mathbf{r}_2) - \theta(\mathbf{r}_1)]). \quad (10) \end{aligned}$$

The key feature for the present discussion is the spatial dependence contained in the explicit phase difference  $[\theta(\mathbf{r}_2) - \theta(\mathbf{r}_1)]$ . Provided that measurements are

<sup>1</sup>The situation would be different if the ground and vortex states corresponded to the same electronic state.

performed at a fixed time  $t$ , this dependence allows us to determine the existence of vortex motion, in a particularly simple way, as we show below.

### Persistent currents

The hydrodynamic formulation of superfluidity [19] introduces the velocity  $\mathbf{v}_s = (\hbar/M)\nabla\theta(\mathbf{r})$ , where  $M$  is the atomic mass and  $\theta(\mathbf{r})$  is the phase of the superfluid component, in our case the vortex phase measured relative to the ground state phase which was tacitly taken equal to zero. Vortex states are characterized by the fact that the circulation of  $\mathbf{v}_s$  is quantized,

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = 2\pi n(\hbar/M), \quad (11)$$

where  $n$  is an integer. In order to detect the circulation of the vortex, it is sufficient to determine its tangential velocity component  $v_\varphi$ . Hence the detectors can remain on a circle centered on the axis of rotation of the vortex and we have

$$\nabla\theta = \frac{1}{\rho} \frac{\partial\theta(\mathbf{r})}{\partial\varphi} \hat{\mathbf{e}}_\varphi, \quad (12)$$

where  $\hat{\mathbf{e}}_\varphi$  is the unit vector tangential to the radial direction. For small distances  $|\mathbf{r}_2 - \mathbf{r}_1|$ , the relative phase appearing in the last term in the correlation function (10) becomes

$$\theta(\mathbf{r}_2) - \theta(\mathbf{r}_1) \simeq (\nabla\theta) \cdot \hat{\mathbf{e}}_\varphi d\rho = (M/\hbar)v_\varphi(\varphi)d\rho. \quad (13)$$

For a general phase variation  $\theta(\mathbf{r})$  the local velocity, and hence the current of atoms, will vary azimuthally. However, for a vortex of topological charge  $2m$ , we have  $v_\varphi = 2m\hbar/M\rho$  independent of the azimuthal position of the pair of closely spaced detectors, and  $n = 2m$  in Eq. (11). Hence, moving the pair of detectors along a circle while keeping their distance  $d\rho$  fixed allows one to determine the presence of persistent currents,  $v_\varphi(\varphi) = \text{constant}$ . Detection of this persistent current is a key signature of a vortex state. From the value of this persistent current one could infer the value of  $n$  if all other parameters were known. Next we describe a second measurement which yields the topological charge more directly.

### Topological charge

In addition, it is also possible to carry out a different class of measurements where detector 1 is held at a fixed position relative to the vortex core, while detector 2 is moved on a circle relative to detector 1. In that case,  $G^{(2)}$  will exhibit oscillations as the relative azimuthal angle ( $\varphi_1 - \varphi_2$ ) between the two detectors is varied, the phase difference being

$$\theta(\mathbf{r}_1) - \theta(\mathbf{r}_2) = 2m(\varphi_1 - \varphi_2). \quad (14)$$

Thus, the second-order correlation function shows interference fringes as the relative azimuthal angle of the detectors is varied, and the topological charge of the vortex may be extracted as twice the number of (complete) bright fringes in the interference pattern as detector 2 is moved through a full circle. Note that in contrast to the preceding scheme, which requires knowledge of the system parameters, e.g. the atomic mass, to determine  $v_\varphi$  absolutely, the present method reveals the topological charge of the vortex,  $2m$ , from a global property of the interference pattern, the number of bright fringes.

To summarize, we have discussed a measurement scheme that permits to fully characterize vortex states in low density atomic condensates, yielding both direct evidence of persistent currents as well as a parameter-free determination of the topological charge.

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